Lab 5

Exercise 1: Find simple functions f (like n, or n^5, or $n \log_2 n$) for each of the functions below so that they are $\Theta(f)$. For the recurrences T below, assume $T(n) = 1$ for $n \leq 2$, and otherwise satisfies the recurrence given.

- n^3
- $.5n^3$
- $10n^7$
- $\sum_{i=0}^{n} i^2$
- $\sum_{i=0}^{n} i^3$
- $\sum_{i=0}^{n} \sqrt{i}$
- $\sum_{i=1}^{n} \sqrt{i} \cdot \log_2 i$
- $T(n) = T(n-1) + 5$
- $T(n) = T(n-2) + 2n$
- $T(n) = T(\sqrt{n}) + 1$
- $T(n) = 2T(n/2) + \log_2 n$

Exercise 2: Recall the Fibonacci recurrence

$$\text{fib}(n) = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1 \\ \text{fib}(n-1) + \text{fib}(n-2) & \text{otherwise} \end{cases}.$$

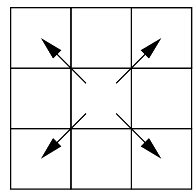
Find a value c so that $\text{fib}(n) \leq c^n$. Prove that this is so using induction.

Exercise 3: Show by induction that every integer 2 or greater is a product of primes.

Exercise 4: Suppose a country only has 3-cent and 5-cent coins. Show by induction that you can make change for any monetary value which is at least 8 cents.

Exercise 5: Show by induction that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$.

Exercise 6: A robot starts off in an infinite grid of cells, at the location $(0, 0)$. At each time step he can move diagonally to the topleft, topright, bottomleft, or bottomright (see the picture below).



Can the robot ever reach the cell (0,1)? Either show a way he can, or show that he can't using induction.